COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Reply to "Comment on 'Small-scale intermittency in randomly stirred fluids' "

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(Received 16 June 1995)

Following the comment [E. V. Teodorovich, Phys. Rev. E **52**, 2118 (1995)], we point out that the Ward identity does not eliminate the possibility of a scale-dependent renormalized vertex when no external momenta are set equal to zero. Assuming a homogeneous scaling in the renormalized vertex, and using Lee's corrected diagrams [Ann. Phys. (N.Y.) **32**, 292 (1965)], dimensional analyses in the self-energy and energy integrals give rise to a correction to the Kolmogorov exponent, indicating the phenomenon of intermittency. However, higher order structure functions are not represented properly due to the Gaussian nature of the model.

PACS number(s): 47.27.Gs

In the paper "Small-scale intermittency in randomly stirred fluids," [1] we attempted to explain the occurrence of intermittency correction to the Kolmogorov scaling. This was achieved by considering the renormalization of the interaction vertex $-(i\lambda_0/2)P_{ijl}(\mathbf{k})$ into a more general vertex. This general vertex is expected to have a form $-(i/2)\Gamma_{ijl}(k|k_1,k_2)$, with $k_1+k_2=k$. We could then expect that the renormalized coupling $\lambda(k|k_1,k_2)$ scale as

$$\lambda(k|k_1,k_2)\sim k^{\sigma}f(k_1/k,k_2/k) \ . \tag{1}$$

Assuming this kind of homogeneity, and carrying out the dimensional analysis in the self-energy and energy integrals, a correction to the Kolmogorov exponent is obtained in terms of σ. Higher order structure functions also show deviations from Kolmogorov-like scaling. However, the corrections go linearly with respect to the order of the structure function (in contrast to the multifractal scaling, yet to be understood theoretically).

Recently, an objection was raised by E. V. Teodorovich [2] against the above scheme, based on the Ward identity

$$\Gamma_{ijl}(k|k,0)=-k_l\frac{\partial}{\partial\omega}G_{ij}^{-1}(k,\omega) \ . \tag{2}$$

This identity, with one of the external momenta set equal to zero in the general vertex, implies that $\Gamma_{ijl}(k|k,0)$ be proportional to $P_{ijl}(k)$, and therefore there is no scaling in the coupling constant $\lambda(k|k,0)$.

We would like to point out that the above Ward identity does not eliminate the possibility of the scaling as in Eq. (1) with $\sigma\neq 0$, where none of the external momenta is zero. Renormalized correlations and higher order structure functions do, in fact, involve the more general vertex $\Gamma_{ijk}(k|k_1,k_2)$ with $k_1\neq 0$ and $k_2\neq 0$, instead of the vertex $\Gamma_{ijk}(k|0,k)$, and, as is well known, higher order structure functions are increasingly important in the phenomenon of intermittency. However, our naive dimensional analysis yields only linearly increasing exponents, instead of a multifractal. Nothing more can be expected from a randomly stirred model, where the external noise statistics is assumed to be Gaussian, so that higher order structure functions are expressible in terms of two-point correlation functions.

In our dimensional analysis we used Wyld's renormalized diagrams. Lee [3], however, suggested the following correction to the renormalized Wyld's diagrams: the left renormalized vertices in the renormalized diagrams should be replaced by the bare ones (in order to avoid some overcounting that is present in Wyld's perturbation treatment). The velocity field was scaled by a factor of k^{σ} in the paper. In order to get intermittency correction, this scaling is, however, unnecessary (cf. the note in Ref. [21] of Ref. [1]). Taking Lee's corrections into account and working with the unscaled velocity field, Eq. (7) becomes $z=2-y/3+\sigma/3$ and Eqs. (10) and (11) become $E(k)\sim k^{1-2y/3-\sigma/3}$ and $\Pi(k)\sim k^{4-y}$, respectively. Requiring a conserved transfer of energy amounts to setting $y=4$, which readily gives the intermittency corrections as $\omega(k)\sim\epsilon^{1/3}k^{2/3}(kL)^{\sigma/3}$ and $E(k)\sim\epsilon^{2/3}k^{-5/3}(kL)^{-\sigma/3}$ [which are the modified Eqs. (12) and (13)]. However the randomly stirred model does not properly represent the higher order correlation functions. Considering the above modifications, Eq. (15) modifies to $\langle|\Delta_r\mathbf{u}(\mathbf{x})|^n\rangle\sim\epsilon^{n/3}r^{n/3}(r/L)^{-n\sigma/3}$.

[1] M. K. Nandy, Phys. Rev. E **48**, 1015 (1993).
[2] E. V. Teodorovich, Phys. Rev. E **52**, 2118 (1995).
[3] L. L. Lee, Ann. Phys. (N.Y.) **32**, 292 (1965).